

The purpose of vector diagrams is to provide a simple visual representation of situations involving complex quantities. The author argues that the present system of vector representation confuses rather than clarifies, and proposes a new method, devoid of arrows, that is claimed to be simpler and less ambiguous. The author is an independent consultant.

A new deal for vector diagrams

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Of all the forms of expression used in electrical engineering, the so-called vector diagrams are perhaps the least satisfactory. The name itself is wrong, since the lines composing the diagrams are not vectors in the strict sense and can, in fact, represent scalar quantities such as potential difference; but let us charitably assume that all teachers take care to make that clear from the start. And the conventions commonly used in contemporary textbooks—and therefore presumably on contemporary blackboards—are anomalous, confusing, uninformative and inelegant.

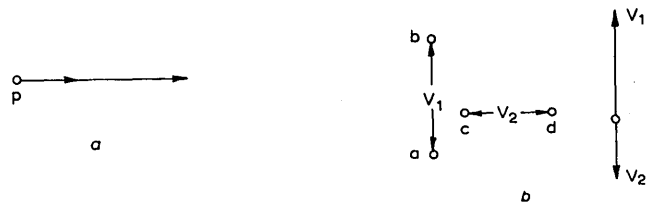
Each vector (from now on the word is to be understood in its accepted inaccurate sense) is drawn as an arrow. In all other contexts this would be understood to indicate longitudinal movement. The first thing the student has to do is to dismiss that medieval idea from his mind and interpret the arrowhead as indicating rotary movement of the line around its far end as centre. He will soon learn also that, when two or more arrowheads meet at a point, they make a confused mess, and that when vectors in phase are added end-to-end, as in Fig. 1*a*, they may legitimately be interpreted as both starting from point P, which is not correct. But his teacher is unlikely to tell him that all the arrowheads, with these disadvantages, are totally unnecessary and serve only to confuse matters and destroy the generality and uniqueness of the diagram.

Confusion in present system

Vector diagrams are supposed to show the electrical conditions in a.c. circuits. In order to do this clearly and unambiguously, there must be satisfactory conventions to relate the vector diagram to the circuit diagram. In most textbooks the voltage symbols on circuit diagrams are accompanied by two oppositely pointing arrows, as in Fig. 1*b*. This practice does at least show the circuit points between which the voltages exist, but that is the most that can be said for it, and it is not an exclusive advantage. The vector diagram in Fig. 1*b* tells us that V_2 is opposite in phase to V_1 , but when we try to apply this information to the circuit, we have a choice of two opposite interpretations and a fifty-fifty chance

of being right. To sportsmen that may be magnificent but it is not electrical engineering.

Curiously enough, books that use these two-way voltage arrows—and they include the works of most



1 Ambiguities of the common notation

a are these vectors consecutive or concurrent?

b a spin of a coin is necessary to determine (for example) the polarity of *c* with respect to *d* when *b* is positive to *a*

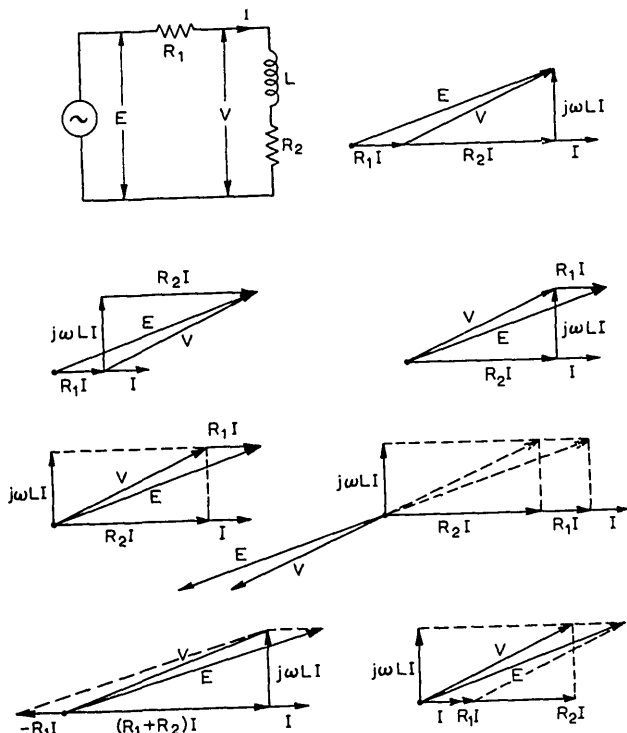
of being right. To sportsmen that may be magnificent but it is not electrical engineering. Curiously enough, books that use these two-way voltage arrows—and they include the works of most

eminent authorities—usually mark currents with one-way arrows, and so are not even consistent. These authors might reply that, if we were to study the rest of the circuit and apply fundamental principles, we ought to be able to work out the right answers. But, surely, vector diagrams should save us trouble, not give us needless mysteries to solve!

Fig. 2 shows a diagram of a very simple circuit and some of the possible vector diagrams that can arise from it according to existing conventions. The student may well be confused when such a variety of patterns can be used to mean the same thing. And by disseminating such diverse conventions the teachers make the marking of examination papers needlessly difficult for themselves.

But that is not the worst of it. What use are these diagrams to us? Can any of them tell us, for example, the potential of the RL junction relative to the LR_2 junction at any chosen phase of the e.m.f.? They cannot. Because of the ambivalence of E in the circuit diagram, we cannot tell which direction the author of the diagram has chosen to regard as positive. Neither, owing to the absence of any arrow relating to the voltage across L , can we tell his intentions concerning that either. You may say that in this case the answer is obvious. But only

because you know already, by applying basic principles to the circuit diagram. If the vector diagram fails to give a clear answer in such a trivial case, how can it be expected to guide one infallibly in situations where there



2 Simple a.c.-circuit diagram drawn according to present conventions, and some of the ways in which its associated vector diagram might be drawn

are perhaps several sources of e.m.f., several current branches, mutual inductance, negative feedback and other complications?

A striking example of the inadequacy of the conventional circuit diagram can be found in a reputable journal for teachers. The ordinary rules had completely broken down under the strain of coping with the simplest possible circuit incorporating a triode valve, and the diagram was incapable of answering the most elementary questions correctly.

Evidence of the confusion resulting from this lack of a clear definitive system of conventions was provided when a class was asked what, in Fig. 3, would be the polarity of the voltage from c to d relative to that from a to b . Eleven said it would be the same and 12 the opposite!

Unambiguous system

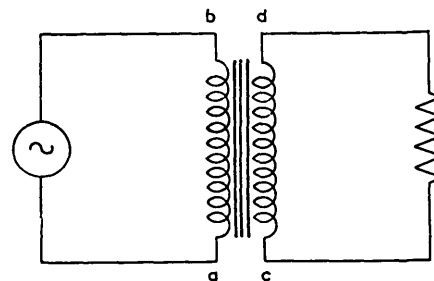
The proposed system, which, it is claimed, is free from all the shortcomings described, can perhaps most conveniently be illustrated by comparing Fig. 4 with Fig. 2. Note that there is not an arrow anywhere. Yet ambiguity is replaced by complete certainty. The vector diagram in Fig. 4 is not only simpler in its conventions and tidier than any of those in Fig. 2; it is the only possible form applicable to the conditions to which it refers. It makes no assumptions as to which part of the

circuit, if any, is earthed, or in fact any assumptions whatever beyond the conventions on which it is based. But it can at once be specialised to take into account the earthing point if known. The potential and phase relationship of any point in the circuit relative to any other point can be seen at once by turning the diagram to the appropriate angle. And it is applicable to the most complicated circuits, with correspondingly greater realisation of its advantages. In fact, it enables clear diagrams to be drawn for circuit situations for which no textbook dares to show conventional vector diagrams, because of the utter confusion that would result.

There is no need to mark voltages or currents on either diagram. Voltages are referred to, when necessary, by the double-subscript notation; e.g. V_{ab} . This notation is, of course, well known, but it has two alternative interpretations. V_{ab} can mean the voltage of a with respect to b , i.e. $V_a - V_b$. Or it can mean the voltage in the direction $a \rightarrow b$, i.e. $V_b - V_a$. The former seems to be the more popular, but it logically requires one to stand on one's head, as we shall see, and, like Father William, most of us ought to be too old for that. The latter usage fits in perfectly with certain universally accepted conventions and therefore has been adopted for the present purpose.

One of these conventions is that, in vector diagrams, 'up' means positive and 'down' means negative. So the natural meaning of the vector dc in Fig. 4 is that c in the circuit is positive with respect to d . Again, if we wanted to refer to that vector in its upward sense, we would call it dc , not cd . The reverse of either of these usages would be unthinkable. A third unthinkable thing is that the vector dc should represent the voltage V_{cd} . We are therefore logically bound to interpret the vector dc as showing V_{dc} at its maximum positive value, c in the circuit being positive with respect to d , and therefore V_{dc} represents the change of voltage in the direction $d \rightarrow c$.

The same symbolism states the undoubted fact that d is negative with respect to c . The usual arrowheaded vector arbitrarily picks out one of these statements.



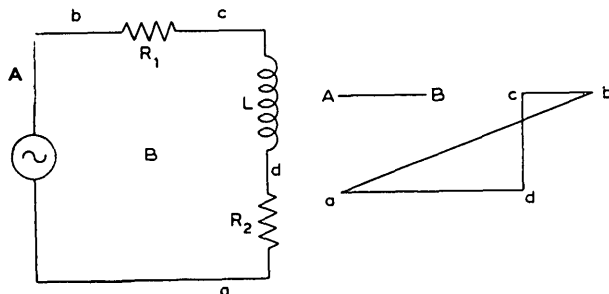
3 Even this simple circuit confuses students taught by currently accepted methods

Not that that is necessarily wrong. There may often be a good reason; for instance, if we knew that c in the circuit was earthed, its potential would be regarded as permanently zero and it would be V_{cd} (at this instant, peak negative) that would concern us, not V_{dc} . The natural way to show this is to make c the point in the vector diagram around which it revolves, and this can

be indicated simply by a prominent dot at c or a small circle around it. At the same time this convention does not confuse the issue by drawing the vector ab radiating from the same point as those representing earthed voltages, but clearly indicates the potentials of a and b (and any intermediate point) with respect to earth.

Notation for current

With regard to current notation, there is at present a complete lack of system. One practice is to number currents arbitrarily. In the simple Maxwell bridge, there are seven distinguishable currents, and therefore 5040 different ways of numbering them, none of which has



4 Circuit and vector diagrams recommended in place of those of Fig. 3

any inherent meaning. Another practice is to use subscript letters and numbers indicating the component through which the current passes, but this too is unsystematic, because some currents go through more than one component. Others again mistakenly imitate the subscript notation for voltages and run into trouble when there is more than one current path between two marked points.

The reason for the failure of the last idea is that the principle of duality is ignored. The analogue of circuit nodes for voltage is circuit meshes for current. Therefore currents should be identified by marking the meshes, just as the nodes are marked for voltages, as already described, but preferably with distinctive symbols such as capital letters. This has been done in Fig. 4, where there is only one current I_{AB} ($= -I_{BA}$). As there are no existing conventions for this, one has to be established, such as that suggested in Fig. 5a, which is easily remembered. It is surprising that this notation has remained undiscovered for so long, when Bow's similar notation for structures has been in use for nearly a century. Its only disadvantage seems to be that, in the comparatively rare a.c. circuits that cannot be drawn without crossing wires (all that can be so drawn should be), there is a difficulty in marking the meshes, but this can be overcome by introducing a 1 : 1 ideal transformer.

One of the incidental features that distinguishes the recommended notation as a system and not just as a makeshift is that, in accordance with Kirchhoff's laws:

$$V_{ab} + V_{bc} + V_{ca} \equiv 0$$

and
$$I_{AB} + I_{BC} + I_{CA} \equiv 0$$

and similarly for any number of voltages and currents.

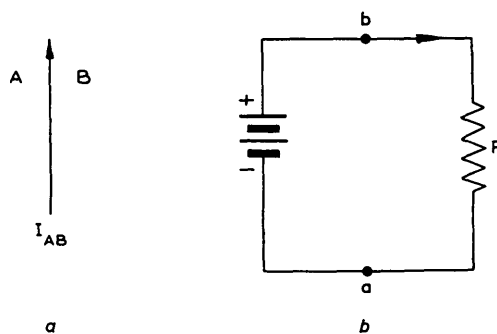
The current diagram is drawn separately from the voltage diagram, so removing one cause of confusion, but it would normally be placed where a current is clearly parallel to some voltage with which it is in phase.

The question of phase

In that connection, one has to be quite sure about how being in phase is indicated by the recommended notation. It seems reasonable to regard the positive direction of an e.m.f. as the direction in which the e.m.f. tends to drive positive current. In Fig. 5b the e.m.f. of the battery tends to drive current (and is succeeding in doing so) through itself from a to b ; so V_{ab} in the recommended notation is positive. This current flows through R from b to a ; so, when considering it in relation to the applied e.m.f., there might be a tendency to call the applied e.m.f. (in the same notation) V_{ba} . This would lead to $V_{ba} = V_{ab}$, which is contradictory, as b is definitely positive with respect to a . So, when considering the phase of the current through any circuit element, one must regard the applied e.m.f. as coming from the rest of the circuit as source. Thus I_{AB} , flowing through R_1 in Fig. 4 from left to right, is by basic principle in phase with the applied e.m.f. V_{cb} , reckoning it from c via a to b . The vector cb is accordingly drawn parallel to AB , which was laid down first at any desired angle.

Alternatively, and in accordance with Newton's third law, one can regard V_{cb} via R as the back voltage to which the resistance R gives rise. Either way the diagram is the same.

How about flux? The basic fact about it is that it is in phase with the current causing it, so it can be identified



5 Suggested current notation

a notation
b diagram to clarify application of notation

by the same notation: Φ_{AB} , for example. A further economy of effort conferred by the advocated system is that there is no need to show flux vectors, as the current vectors serve for both. If we want to call attention to the fact that a certain current is responsible for a significant flux, we can mark the appropriate current vector with a Φ .

It is felt that the system described can offer, in the teaching of basic a.c.-circuit principles, a degree of logicity, simplicity and elegance that is at present sadly lacking. It is hoped that enough has been said to draw forth either a valid defence of the present teaching, or, failing that, a change.